

An Experimental Analysis of Heuristics on Time Complexity for Permutation Flow Shop Scheduling

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Abstract—Flow shop scheduling is an important decision making process in which all jobs has same processing order on all given machines. Hence, there is real sequencing order of machines. Better sequencing and scheduling system has a significant impact on time complexity. Since, problem is NP-hard for more than two machines, effort has been made in the progress of heuristic procedures so that optimal solutions can be procured in less time interval. In this paper, a review and comparison has been made on time complexities of heuristics in flow shop scheduling environment and then future research issues regarding reducing time complexity has been considered.

Keywords: Sequencing, Scheduling, Johnson Algorithm, Palmers Algorithm, CDS algorithm, NEH Algorithm, Permutation Flow Shop Scheduling, Time Complexity.

1. INTRODUCTION

Scheduling is a decision making process which is used in many service industries, planning and manufacturing system. Scheduling process arises when resource availability is fixed by prior planning decision [17]. There are four scheduling approach by which scheduling has reached as a systems approach: formulation, evaluation, synthesis and analysis. Its target is to optimize some known objective. Since, each action is in competition with other, scheduling of job is neither straight forward nor effortless. Basically scheduling focus on two important results [16]:

- 1) Sequences of jobs that will be processed by two or more machines.
- 2) Loading of schedules by machines which identify sequential arrangement.

Sequencing and scheduling both the terms are used interchangeably but there is a difference between them which should be known. Sequencing refers to an arrangement of events or item in one fixed order [1]. There are many types of sequencing problem which is used in industries,

manufacturing systems etc such as first in first out, processing time basis, job size basis and precedence basis. Scheduling is a decision making process and it bothers with allocation of resources [2]. First publication on scheduling has appeared many years ago. But initial research on flow shop scheduling problem was done by Johnson [3]. Johnson's algorithm is the primary algorithm that yields optimal result for two machines and n jobs flow shop scheduling problem. Later many algorithms such as beam search and branch and bound were developed to give exact solution for above problem. But problem for m-machine and n-job is NP-hard therefore, optimal solution techniques are considered. In recent years, many heuristic algorithms such as CDS (Campbell et al) [10], Palmer's [18], NEH (Nawaz et al) [11] have been developed in flow shop scheduling so that optimal solutions can be obtained and time complexity can be minimize [4].

2. NOTATIONS AND FRAMEWORK

A scheduling problem is denoted by a triplet that is, $\alpha|\beta|\gamma$ [9]. The α -field contain single entry and it describe machine environment. The β -field contain single entry, multiple entry or no entry and it tell us about processing characteristics whereas γ -field describe the objective which has to minimize in the problem and contains single entry. In flow shop scheduling subscript 'i' refers to machines while the subscript 'j' refer to jobs and pair (i,j) refer to operation or processing step. Processing time (p_{ij}) [12] refer to processing time of job 'j' on machine 'i'. Release date (r_j) [9] it refers to time when job appears at the system or we can say that ready date of job 'j'. Permutation (prmu) [9] it is a constraint which arrives in flow shop scheduling environment in which queue in front of each machine may appear in first-in-first-out (FIFO) manner that means the same order of jobs is maintained throughout the system. Make-span (C_{max}) [9] it is defined as completion time of last job when it leave the system. There are many heuristic

which focus on reducing make-span so that optimal utilization of machine can be done.

3. FLOW SHOP SCHEDULING PROBLEM (FSSP)

The classical method of flow shop scheduling problem (FSSP) is one of the most fascinating areas of exploration for more than 50 years [5]. Flow shop scheduling problem is a planning procedure in which n-jobs have to be schedule on m-machine in same sequence order [15]. There are many algorithms developed to solve this problem and in most of them same assumptions have been taken regarding the environment of sequencing. Some assumptions which are considered in any flow shop scheduling environment [2].

- Machines are always available throughout the scheduling period.
- Order of sequence in all machines is same throughout the scheduling operation.
- Preemption is not permitted i.e. once an operation begins on the machines it should be completed before other operation can start on that particular machine.
- All processing time of jobs on machine should be known, in advance.
- A set of n jobs are available at time zero for processing.
- Each job will be processed by each machine once and only once. Job does not become accessible to next machine when processing of current machine is going on.
- Inventory is allowed in this process.

4. JOHNSON’S ALGORITHM

Johnson (1954) [3] was one of the earliest heuristic known in which two machines is considered in flow shop scheduling problem with the objective of minimizing make-span. Many research papers have been developed on Johnson’s algorithm and its extension so that better solutions can be obtained. Its significant result has become standard theory of scheduling. Johnson’s rule is as follow [14]:

- a) Minimum process time for machine 1 must be greater than or similar as that of maximum process time for machine 2.
- b) Minimum process time for machine 3 should be greater than or similar as that of maximum process time for machine 2.

Step by step explanation:

- 1) Let set a = {j, $T_{i1} < T_{i2}$ }
- 2) Let set b = {j, $T_{i1} \geq T_{i2}$ }
- 3) Sort the jobs in ‘a’ by ascending order of T_{i1}
- 4) Sort the jobs in ‘b’ by descending order of T_{i2}

- 5) Then merge both the sorted order that is, {a} followed by {b}

5. PALMER’S HEURISTIC ALGORITHM

After Johnson algorithm many researchers developed distinct heuristics for m-machines flow shop scheduling problem so that optimal solutions can be obtained [3]. Palmer (1965) [18] developed a slope index method to calculate sequences of jobs on machines. In this method a weighted sum of each job is calculated and priority was given to those jobs whose processing time tends to increase from one machine to other.

It has two steps by which optimal solution can be obtained

- 1) For m-machine and n-job, we will calculate slope index ‘Nj’ for j^{th} job

$$N_j = - \sum_{i=1}^m \{m - (2 * i - 1)\} P_{ij} \tag{1}$$
- 2) Now after calculation of ‘Nj’ sequence the jobs in decreasing order.

6. CAMPBELL, DUDEK AND SMITH HEURISTIC ALGORITHM

CDS is an extension of Johnson’s algorithm which is applied to m-machine and n-job problem. In this method at most (m-1) different sequences are developed and from those sequences best sequence is identified [10]. The main emphasis of this heuristic is to reduce make span in flow shop problem [19].

Algorithm is as follow

- 1) Create additional number of n-job and m-machine problems, P, where $P \leq m-1$
- 2) For first problem set $K=1$
- 3) Now calculate total processing time for each job (i) on machine-1 and machine-2

$$M1 = \sum_{j=1}^K T_{ij} \tag{2}$$

$$M2 = \sum_{j=m-k+1}^m T_{ij} \tag{3}$$

- 4) Now apply Johnson rule to each (m-1) sequences. Select the minimum processing time from two column matrix.
- 5) Increment $K=K+1$ and repeat until $K=P$
- 6) Choose the minimum total processing time out of them and that will be the best sequence.

7. NAWAJ ENSCORE HAM HEURISTIC ALGORITHM

NEH [11] is considered as the finest heuristic algorithm when compared with other simple heuristics by Taillard [7], Turner and Booth [6], and Ruiz and Maroto [8]. In this algorithm insertion technique is applied and enumeration will be $[n(n+1)/2]-1$. Steps for NEH algorithm is as follows:

- 1) Compute the sum of processing time of each job
- 2) Then order the jobs in decreasing order of processing time on machines
- 3) Then take first two jobs, as if there were only these two jobs and schedule them
- 4) Repeat step 5, for $k = 3$ to n
- 5) To minimize the partial make-span, we will insert K^{th} job at the place.

8. ANALYSIS OF HEURISTIC ALGORITHMS

As all are informed that most of the heuristic algorithms study only processing time and give results which are based on using separate approaches. Heuristics are approximation methods and flow shop scheduling problem is NP-hard for m-machine problem, so it cannot be stated that heuristic algorithm which we are using, always give us optimal/near optimal solution [22]. For two machine problem in flow shop environment Johnson's algorithm is the best and exact heuristic algorithm [21] to provide optimal solution. Johnson also provides its rule to apply for three machines problem but no efficient solution was produced. For three machines flow shop problem there are no exact methods which are known. The problem which is looking very simple converts into very complicated one when machine exceeds three [21]. Later many heuristic algorithms have been developed to provide optimal/near optimal solutions when machine exceeds three. Palmer's algorithm [18] developed a slope index method by which sequences of jobs is calculated. Palmer performs better when there are small computational times [23]. CDS is an extension of Johnson's algorithm [10]. In this heuristic (m-1) sequences are developed and from those best one is chosen. For m-machine problem and for large computational time it performs better and gives near optimal solutions than Palmer or Johnson [24]. NEH [11] is considered as the finest polynomial heuristic algorithm when compared with other simple heuristics by Taillard [25], Turner and Booth [7], and Ruiz and Maroto [8]. It produces more elaborated results than other heuristics so it gives best optimal solutions as compared with other. With this analysis of different heuristics complexity level have been studied by Taillard [7] and are described below

9. PERFORMANCE MEASURES:

In this paper we are using RPD [28] that is, relative percentage deviation as performance measure so that each algorithm can be differentiate with other algorithm solutions.

The relative percent deviation is given by:

$$RPD = \frac{SH - SR}{SR} * 100\%$$

Where:

SH = solution of heuristic problem

SR = solution of reference problem

Table 1: Heuristic Algorithm and there complexities

	Complexity	RP D	RP D	RP D	RP D	RP D	RP D
Jobs	n	20	20	50	50	100	100
Machine	m	5	10	5	10	5	10
Johnson	$n \log(n) + nm$	9.5	11.2	15.5	16.3	18.4	19.1
Palmer's	$n \log(n) + nm$	8.4	9.2	12.4	13.6	14.2	14.9
CDS	$nm^2 + n \log(n)$	4.5	5.1	8.5	9.7	10.5	11.0
NEH	n^2m	2.1	2.0	3.8	4.1	5.2	5.1

NEH appears to be best heuristic algorithm and CDS is at second place also performs better and provide optimal solution [27].

10. CONCLUSION AND FUTURE WORK

An important issue for heuristic algorithm is the quality of the solution and the time required to acquire such solution. In this paper we are concerned with the analysis of time complexity. However, such a trade-off is seen as a target programming issue [26]. Decision makers want to have finest quality of the solution. From table-1 we conclude that NEH is the best heuristic algorithm. It gives us best solutions in more elaborated way. It takes $[n(n+1)/2-1]$ [11] enumeration. Whereas CDS make (m-1) [10] number of sequences and is at second place in terms of complexity and it also performs better. Johnson [14] is optimal for two machine problems and provides better solution than others. Palmer is good when short computational times are needed [18].

In future work we will be considering CDS algorithm as our benchmark algorithm because it uses (m-1) [10] sequences to give optimal solution but when machine increases, complexity also increases. An attempt will be made to study about sequence dependent operation and to reduce time complexity by reducing number of sequences.

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